Speed Bumps – Calming or Threatening Traffic?

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Abstract

In this paper the problem of speed bumps ('sleeping policemen') will be analyzed taking the dynamic aspects of vehicle motion into account. A simple mathematics simulation vehicle model is presented and afterwards verified by real driving experiments. It turns out that ramps with lengths (in driving direction) that are shorter than the vehicle's wheelbase can lead to a critical dynamic minimization of vehicle ground clearance.

1 Introduction

To enforce speed limits, especially in residential areas, in front of schools, retirement homes and hospitals, speeds bumps are considered by some to be an adequate means. Usually, these are built as a short elevation of the road level that forces the driver to reduce speed in order to avoid physical discomfort or damaging his vehicle. Nevertheless, ground contacts of the vehicle undercarriage may be observed in spite of obeying the posted speed limit. In many cases, this leads to a protracted law suit between vehicle owner and municipal administration, considering the liability for the vehicle damages. In Germany, there are no compulsory regulations in regard to the design of speed bumps. Solely the recommendations on the conception of residential streets include a design template [1].

Quite often, municipal administrations rely on their own design concepts when establishing speed bumps. These may undergo driving tests that lead to subjectively rating in respect to safety and efficiency. In order to do a thorough job on this, the authorities would need detailed knowledge on vehicle dynamics. The vast amount of vehicles that are damaged although the driver paid attention to the posted speed limit clearly shows that most often, vehicle dynamic behavior is not adequately taken into account. In the following, we shall therefore put a spotlight on the driving dynamics when passing over a speed bump.

2 Mathematical modeling

For the description of the body motion we use a single track model according to **fig 1**. The body with mass m and pitch moment of inertia Θ is coupled to the massless axles via spring damping systems that are also considered as massless. As the natural frequency of the vehicle body is ten times lower than that of the axles oscillating on the tires [2], axle motion follows the ground profile at low driving speeds, while the body may oscillate on the spring damping system. The tire damping will be neglected in the following so that the axles exactly follow the ground profile; this assumption implies exclusion of tire take-off.

Notation:

l	wheelbase
ψ	distance between the center of mass and the front axle normalized by the wheelbase
u_1, u_2	elevation of wheel standing points in respect of the normal road level
x_1, x_2	excursion of the vehicle body at the fixation points of front and rear axles in respect to the normal position

A force balance then leads to the dependency

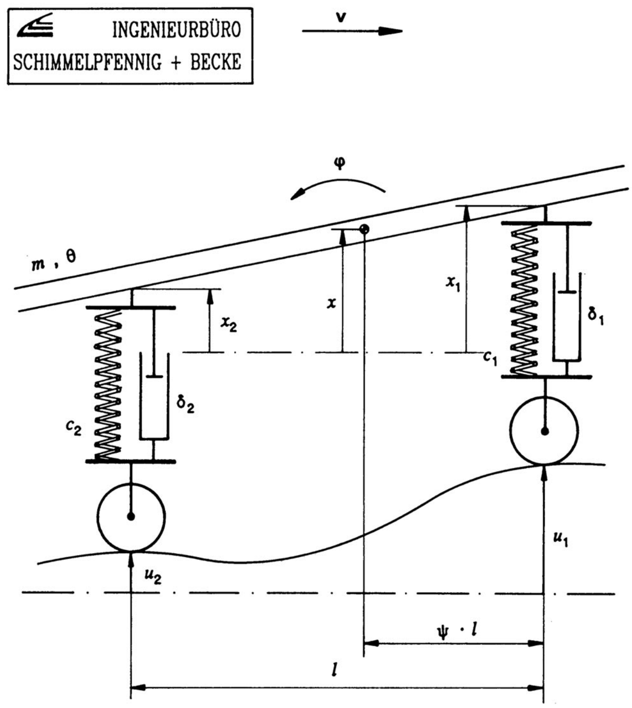

Fig. 1: Mechanically simplified vehicle model

$$m \cdot \ddot{x} = c_1(u_1 - x_1) + c_2(u_2 - x_2) + \delta_1(\dot{u}_1 - \dot{x}_1) + \delta_2(\dot{u}_2 - \dot{x}_2) \tag{1}$$

Analogously, balancing the torque will lead to

$$\frac{\theta\,\ddot{\varphi}}{l} = \psi\ c_1(u_1 - x_1) - (1 - \psi)\,c_2(u_2 - x_2) + \psi\ \delta_1(\dot{u}_1 - \dot{x}_1) - (1 - \psi)\,\delta_2(\dot{u}_2 - \dot{x}_2) \tag{2}$$

With the geometric relationships

$$x = x_1 + \psi\,(x_2 - x_1) \tag{3}$$

$$\varphi = \operatorname{artan}\left(\frac{x_1 - x_2}{l}\right) \approx \frac{x_1 - x_2}{l} \tag{4}$$

and the definition

$$\tau = \sqrt{\frac{\theta}{m}} \cdot l \tag{5}$$

we will arrive at the following differential equation system for the description of body motion

$$(1 - \psi)\ \ddot{x}_1 + 2\ d_1\ \omega_{0_1}(\dot{x}_1 - \dot{u}_1) + \omega_{0_1}^2(x_1 - u_1) + \psi\ \ddot{x}_2 + 2\ d_2\ \omega_{0_2}(\dot{x}_2 - \dot{u}_2) + \omega_{0_2}^2(x_2 - u_2) = 0 \tag{6}$$

and

$$\frac{\tau^2}{\psi}\ddot{x}_1 + 2\ d_1\ \omega_{0_1}(\dot{x}_1 - \dot{u}_1) + \omega_{0_1}^2(x_1 - u_1) - \left\{\frac{\tau^2}{(1 - \psi)}\ddot{x}_2 + 2\ d_2\ \omega_{0_2}(\dot{x}_2 - \dot{u}_2) + \omega_{0_2}^2(x_2 - u_2)\right\} = 0\,. \tag{7}$$

Eqs. (6) and (7) make use of the common abbreviations

$$\omega_{0_i} = \sqrt{\frac{c_i}{m}} \tag{8}$$

and

$$d_i = \frac{\delta_i}{2\ m\ \omega_{0_i}}\,. \tag{9}$$

Eqs. (6) and (7) build up a set of two coupled linear inhomogeneous differential equations of second order that have to be solved in respect to the input signals u_1 and u_2.

The input signal $u_1(t)$ depends on the geometry of the speed bump $u_1(s)$ and the vehicle speed

$$u_1(t) = \frac{1}{v}u_1(s)\,. \tag{10}$$

The input signal $u_2(t)$ at the rear axle is identical to the input signal $u_1(t)$ at the front axle but delayed by a time lag caused by the wheelbase l

$$u_2(t) = u_1\left(t - \frac{l}{v}\right). \tag{11}$$

3 Simulation of body motion

Although the homogeneous part of the differential equation system eqs. (6) and (7) may be solved analytically, the solution of the inhomogeneous equation system may only be arrived at by numerical simulations, at least for arbitrary input signals.

The profile segments of the speed bump used in the driving experiments were described by functions for the simulation. Following the data given in [3] we choose the following dataset for our simulations

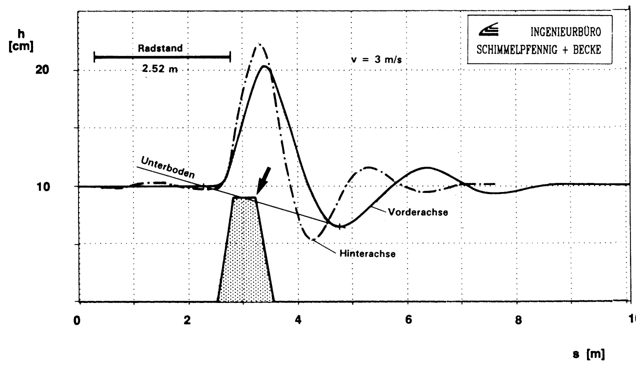

Fig. 2: Body motion at front and rear axle while passing the bump with v = 3 m/s

Explanation of the labels on the graphs:
Radstand = wheelbase
Unterboden = undercarriage
Vorderachse = front axle
Hinterachse = rear axle

$$\psi = 0.42 \qquad \tau = 0.47$$
$$\omega_{01} = 5.0\, s^{-1} \quad \omega_{02} = 6.1\, s^{-1}$$
$$d_2 = 0.256 \quad d_2 = 0.210$$

Figs. 2 and **3** show the result of the simulation runs for two different vehicle speeds. The figures show the trajectories of the vehicle body at the fixation points of the front and end rear axle. As variables x_1 and x_2 just denote the excursion from normal level, the vehicle ground clearance may be chosen arbitrarily. It was set to 10 cm. Looking at the figures, one has to keep in mind that the scaling of x- and y-axis differs.

The illustrations depict that body motion follows the ground profile with a significant delay. When the ground clearance at the front axle reaches its minimum, the according axle is already one meter behind the endpoint of the speed bump. At the same instant, the rear axle has not yet reached the speed bump. By connecting the corresponding fixation points it becomes clear that the undercarriage hits the speed bump in this situation.

Higher driving speed not only decreases the amplitudes of body motion but also shifts the position of the front axle minimum to a point further away from the speed bump, i.e. 3 m. At this instant the rear axle has also passed the obstruction.

Thus we make an observation that might seem strange at first glance: the chicane may be passed without problem at higher speed while the undercarriage will touch it at lower passing speeds.

Partly this is due to the fact that lower speeds reduce the relative motion between body and axles. As this relative motion causes the energy dissipation, the attenuation increases at higher driv-

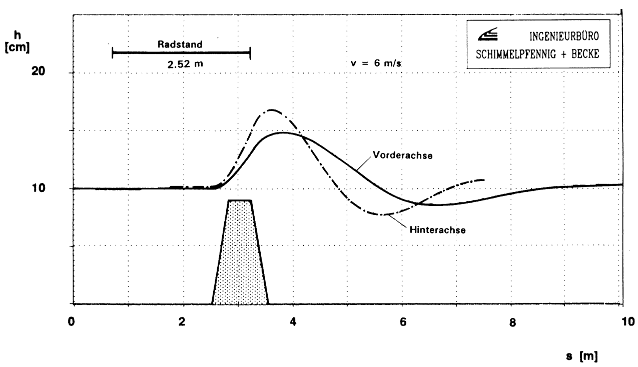

Fig. 3: Body motion at front and rear axle while passing the bump with v = 6 m/s

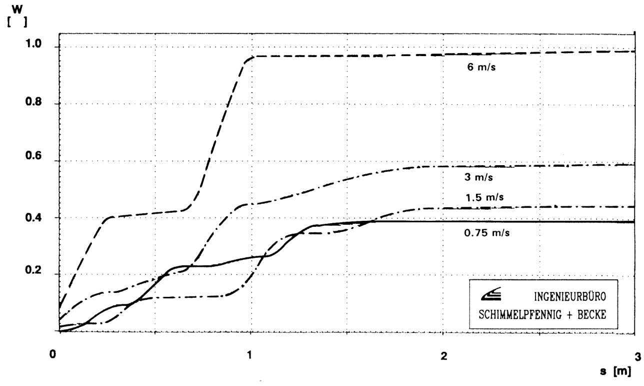

Fig. 4: Friction work at the front axle as a function of driving speed

ing speed. The friction loss may be calculated by

$$W = \int \delta \left| \dot{x}_{rel} dx_{rel} \right| \tag{12}$$

with

$$x_{rel} = x - u \tag{13}$$

and thus

$$W = \delta \int \dot{x}_{rel}^2 d\,t\,. \tag{14}$$

Fig. 4 shows the friction loss at the front axle versus the traversed distance. It was normalized such that a driving speed of 6 m/s lead to a final value of '1'. The energy loss is depicted for a variety of driving speeds.

Especially when doubling the driving speed from 3 m/s to 6 m/s we state a significant increase of the friction work. At lower driving speeds, the impact of driving speed on the friction work is much less. We have to admit, though, that our model is a poor description of vehicle dynamic behavior at low driving speeds because it neglects the effects of dry and adhesive friction.

4 Driving experiments

To verify the results of the simulation, we conducted driving experiments at our test ground with a variety of passenger cars. To do so, we built a speed bump of 9 cm, respectively 7 cm height and 1 m length as depicted by **fig. 5**. The speed bump had two pass-overs with the distance between adjusted to the track of the test vehicle. The gap between the pass-overs was filled with plasticine. Ground contacts could thus be detected without damaging the undercarriage. The profile of the speed bump was made of two ramps of 30 cm length with a 40 cm plateau of constant height in-between.

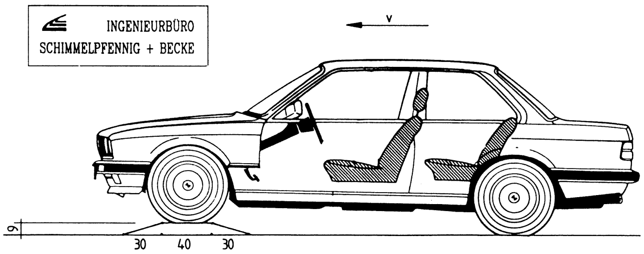

Fig. 5: Test ramp

Generally speaking, all species of vehicles (compact car, middle-class, sporty cars) behaved similarly when passing over the speed bump. If the speed bump was traversed with walking speed, no vehicle had ground contact when loaded with only the driver. Rising the speed to 10 km/h caused ground contact for the lowered sporty car. The contact point was located at the frontal part of the muffler. All other vehicles had no problem when passing the obstruction at this speed. After the speed was raised to 20 km/h, no vehicle, not even the sporty car, contacted the speed bump.

Thus, for the sporty car we found hints on a critical passing speed that could be found for all kinds of test vehicle when loaded with five passengers. While the loaded upper-class vehicle (Mercedes class S, W 129) contacted the speed bump at nearly any passing speed, the other vehicles only showed this behavior in a speed range higher than walking speed and significantly below 30 km/h.

Even lowering the height of the obstacle from 9 cm to 7 cm could not inhibit ground contact for most loaded vehicles. The main contact point always was on the muffler.

5 Conclusions

From our experiments we conclude that there is a critical passing speed for short speed bumps. The exact value of this critical speed depends on the geometry, especially the length of the speed bump and vehicle parameters such as wheelbase, ground clearance and damping.

The general motion pattern may be described like this:

When mounting the speed bump, the front wheels are elevated and the suspension thus is compressed. After a short delay, the front of the vehicle body also rises. Due to pitch oscillation behavior, the rise of the vehicle front must be followed by a downward deflection of the body.

If this deflection occurs when the front wheels have run down the speed bump while the rear wheels have not yet reached the obstacle, we may observe a critical ground clearance between vehicle undercarriage and upper part of the speed bump. If the vehicle ground clearance is rather low, either by default or due to loading, this might result in contacting the speed bump.

If the driving speed is either lower or higher than the critical speed range, ground contact is generally avoided. So short speed bumps may be traversed at walking speed or with speeds as high as 30 km/h without problem, while the vehicle contacts the speed bump in a speed range of 10 – 15 km/h. As such an effect is surely not expected by the normal driver, dynamic vehicle behavior should have a stronger impact on the design of speed bumps.

References

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