Brake Reactions under Mesopic Viewing Conditions at Weak Contrast Stimuli

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Summary
Under illumination conditions typical for inner city pedestrian accidents at night (mesopic perception) the break reaction time of 48 subjects was measured. The contrast between a simulated pedestrian and the background as well as its position in the visual field was varied. The results show an exponential increase of the simple reaction time at low contrasts. If the object is exposed 5° left of the visual axis the reactions times are about 17 ms faster than the reaction times with foveal exposition.

1 Introduction

Reconstructing a traffic accident, the supposed reaction time may affect the avoidance to the same extent as the collision speed. Although the parameters that have influence on the collision speed are often subject of dispute, the reaction time is rarely discussed. This is at least the situation in Germany, where the *20. Deutscher Verkehrsgerichtstag* has posed its recommendations on reaction times in the early 80s.

The recommendations of the *20. Deutscher Verkehrsgerichtstag* consciously left the problem of perceptibility of the reactions stimulus unsettled. The recommendations base on experiments in which the test persons had to react to the onset of the braking lights of a lead vehicle. The reaction stimulus – the braking lights directly in front of the driver – was always clearly visible. But which reaction times should we expect when contrasts get near to the limits of perceptibility as they are sometimes reached in nocturnal pedestrian accidents? Up to now they are only a few investigations on this special question.

In the laboratory experiment presented in this paper we investigated reaction times on faint contrasts. At the lower limit the contrast was 4.5 times over threshold, the reconstructionist would call this a ‘praxis factor of 4.5’. This contrast reaches the upper limit of the contrast domain discussed in nocturnal pedestrian accidents which is typically 3 – 5. This experiment continues our work published in [4]. In this earlier experiment we investigated the influence of contrast and seeing angle on the visual reaction time with the same experimental setup. In the current experiment the contrast of the stimulus was further diminished in 5 consecutive steps.

Beside the influence of contrast the effect of the seeing angle was investigated. Is the reaction time to peripheral oncoming objects longer than that to stimuli appearing in the center of the vision field? And if it is: Is the reaction time to peripheral objects raised to an extent that we have to take the time needed for gaze shift into account?

2 Reaction Types

In [4] we pointed out the difference between discrimination and detection (cf. also the statements proposed by Cohen [13]). In the experiment presented here the vision task consists of simple detection of the reaction stimuli. Considering the question to which extent the results of this laboratory experiment may be applied to traffic accidents we want to reflect the classification of reaction types according to Donders [5] (cf. also Luce [6] and Welford [7]):

- **Simple reaction (Donders type A)**
  There is only one trained motion action which applies to all oncoming stimuli. One example of such a situation in normal traffic is driving in a residential street marked by the traffic sign 'playing kids'. If an object intrudes the driveway under such circumstances, the driver has to react in any case with a block braking maneuver. Otherwise he might loose to much braking time. The driver may not take time to make sure that the oncoming object really is a child. Even if it was only a ball or a dog there is impending danger that a child will follow.
- **Go-NoGo-Reaction (Donders Type C)**
  The reaction stimulus has two different appearances. The reaction (Go!) is forced only by one of the two possible forms. In the other case the reaction should be suppressed (NoGo!). Without doubt the question 'braking or not?', i.e. call motion program 'braking' or not, is one of the most common decisions forced by daily traffic. But it is quite difficult to think of an example where this decision only depends on two clearly distinct forms of the reaction stimuli. You may think of a foreign driver nearing a traffic light that it is excluded from his sight by an obstacle (for instance a commercial car). Suppose that he gains sight at the traffic light when he reaches the stopping distance. In this case the decision 'braking or not?' has to be made instantly according to two clearly distinct forms of the reaction stimulus, i.e. traffic light red or green.
- **Choosing reactions (Donders Type B)**
  The reaction stimulus may has at least two different forms to which the test person has to respond differently. Each form of the stimulus therefore claims for another motion program. If there are really just two different forms of the reaction stimulus, there still is a difference to the Go-NoGo-Reaction. Displaying the 'negative' form of the reaction stimuli the motion program has not only to be suppressed but another motion program has to be called. Typical accident situations often claim for a decision between evasive steering and braking. So depending on the driver's decision two different motion programs have to be called and we face a real choosing reaction.

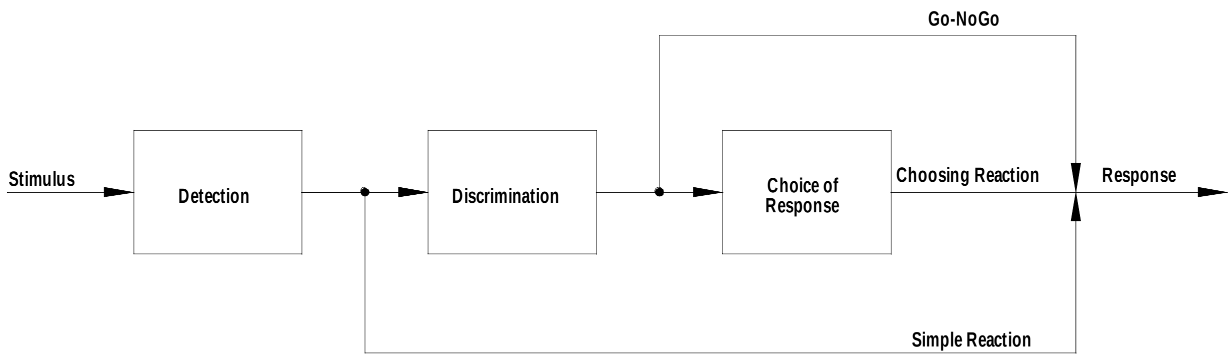

Fig. 1: Model of the reaction process according to Donders

According to Donders' model the simple reaction time has to be the fastest, **Fig.** 1. This case only claims for stimuli detection without any decision on response program. Switching to Go-NoGo-models the reaction time will be prolonged because this situation claims for a discrimination of stimuli. Furthermore, it claims for the decision whether to induce response or not.

The choosing reaction also claims for a discrimination but in addition the test person has to choose between different response alternatives. The dependence of reaction time on the number of alternatives is described the Hick-Hyman-law [6,7]:

$$t_R = a + b \cdot ld\,(N).$$

a and ***b*** are parameters while ***ld(N)*** is the logarithmus dualis. The Hick-Hyman-law therefore describes the reaction time in linear proportion to the information contained in the signal.

The experiment presented here measures the reaction time for a simple reaction (Donders type A). The experiments are therefore limited to situations described in the according paragraph. The reaction times gained by this experiment will be less than those in real traffic. The attentive load posed on the test persons in the laboratory situation is considerably below that of normal traffic. But the qualitative influence of contrast and seeing angle may be transferred to traffic situations.

3 Test Persons and Experimental Setup

The test persons had to react to the simulation of pedestrian as quickly as possible with a braking maneuver. The pedestrian was simulated by a rectangular bar that was projected on a computer screen either straight ahead (foveal perception) or with 5° offset to the right (peripheral perception). The background luminosity was about 0.5 cd/m² which corresponds to that in a nocturnal lightened residential street. The contrast between bar and background was varied in five steps. We evaluated the simple reaction time for stepping on the brake pedal in dependence on that parameter. The reaction time was defined as the time interval between appearance of the pedestrian simulation and touching of the brake pedal. The time intervals between the projections of the reaction stimuli varied randomly while the test persons had to fulfill a concentration task in between.

Fig 2: Experimental setup

The experimental setup was essentially the same as described in [4], but it was optimized at some points (**Fig.** 2). The head of the test person is fixed in a gear in order to control the seeing angle. A black cardboard tunnel between head and computer screen protects the test person from visual irritations coming from outward the experimental setup. During the test the main light was switched off, there was only the darkened lamp that can be seen in the right side of the picture. Below the **Table** you can see the combined gas- and braking pedal. The touching of both pedals is controlled by the experimental computer. The chair that the test person is sitting on is fixed to the ground in order to get a defined position for the legs.

In contrast to the preceding experiment the reaction stimuli are projected each time in the middle of the screen in order to avoid the variation of luminosity outside of its center. This precaution is necessary to get reliably reproducible contrasts. As the reaction stimuli is projected each time at the center of the screen, its position of appearance is fixed. To vary the seeing angle, the gaze of the test persons was led by the concentration task to varying points at the screen, i.e. either to the center or to 5° offset to the right. As described in [4], the concentration task forced the test persons to indicate the opening of a Landolt-pattern. The extent of the opening was adapted such that it was slightly above the visus of the test persons. Therefore, the test persons were forced to fix the pattern in foveal perception in order to solve the task.

The luminosity of the screen background was 0.524 cd/m². The luminosity of the reaction stimuli was varied in five consecutive gray scale steps (0.573 - 0.796 cd/m²). The exact luminosity and contrasts may be seen in **Table** 1. In the last column we calculated the contrast as a multiple of the perception threshold, the so-called 'praxis factor'. The perception threshold may be drawn from the so-called 'Berek-curves' in DIN 5037 (DIN = German Industry Norm). At the luminosity given by the background screen it is about 0.011 cd/m².

Gray step	Luminosity [cd/m²]	Contrast	Praxis factor
85	0.524	0.00	0
86	0.573	0.09	4.5
87	0.625	0.19	9.2
88	0.676	0.29	13.8
89	0.735	0.40	19.2
90	0.796	0.52	24.7

Table 1: Luminosity and contrasts of the reaction stimuli

The test persons consisted of 48 students taking part in two seminars on working psychology. Each test person contributed ten measurements at each combination of the two factors 'contrast' and 'seeing angle', i.e. 100 measurements in a whole. So summa summarum the evaluation of the experiment is based on 4800 measurements.

4 Evaluation Strategies

Before starting with the essential evaluation we want to make same preliminary remarks on the evaluation strategies.

4.1 Variance Generating Factors

In comparison to technical experiments the results of psychological experiments are typically subjected to excessive variances. Therefore, we have to prove whether the observed effects of some influencing factors may have occurred by chance. If this probability is below 5%, the result is called (statistically) significant (by convention). If the probability is below 1%, the result is called highly significant.

In the case under consideration the variance generating factors are:

- Influence of the contrast steps (factor A) σ^2_A
- Influence of the seeing angle (factor B) σ^2_B
- interdependence between the two factors (interaction) $\sigma^2_{A \times B}$
- difference in performance of different test persons (interpersonal differences) σ^2_{TPs}
- difference of performance of one and the same test person (intrapersonal variation) σ^2_{TP}

The results of the experiment may now be evaluated with different aims.

4.2 Influence of the Experimental Factors

Most of all we are interested in the influence of the three first called factors on the reaction time. From that point of view the two other influencing factors are just sources of error that may obscure the ‘true’ result. The influence of the two factors and their interaction therefore has to be compared to the variance of the error sources. Having done so, we may decide whether the influences of the experimental factors are significant according to the above explanation. The variance analysis is a procedure suited to answer such questions [12]. The most calculation procedures for the significance impose that the parameter under examination, in our case the reaction time, underlies a normal distribution. We know from numeral studies that reaction times do not fulfill this condition. Even the reaction times from one and the same test person under the same steady test conditions give a left-steep distribution, for instance a Gamma-distribution with offset [4,6]. Fortunately, the results of a variance analysis are not much affected by a violation of this assumption [12].

On the other hand we know from experience that most scaleable properties of test persons, for instance the height, underlie a normal distribution. If we characterize the performance of a test person depending on contrast and seeing angle with only one number, we may expect that this scaleable property underlies a normal distribution over the different test persons. In this case the standard test for significance may be applied. A suitable parameter to represent a significantly variant measurement like the reaction time is the median which is very stable even under unfortunate conditions. It is not affected by single brake-outs as they are generally observed in reaction time experiments. It therefore approaches to its theoretical limit more quickly than the mean of the reaction times. In contrast to the individual fluctuations of performance, the differences in performance between different test persons as represented by median may be expected to be normal distributed. So it may be investigated by a variance analysis.

So for the test of significance for the ten tested combinations of experimental factors we will use the median. Each median is based on 10 measurements made for each person under this combination of experimental factors. Each person supplies 10 medians in a whole so that we get 480 Medians in a whole. As a test of significance, we use the two factor variance analysis. This procedure checks to which amount the overall variance of the measurements is caused by the different variance generating factors.

If we use a test plan that investigates all test persons under all test factor combinations, the difference of performance between the various test persons may be eliminated as a variance generating factor. Generally, the difference in performance between different test persons affects all test factor combinations in the same manner: The response of a fast reactor is always quicker than the response of a slow reactor, independent of the test factor combination. So we may expect that the individual ability generates a constant offset that affects all reaction times under all combinations of test factors in the same manner. We may characterize this individual ability by the mean of the ten reaction time medians. Subtracting this mean from the ten medians (ipsative measurements) will eliminate personal ability as a variance generating factor.

4.3 Percentiles

On the other hand we want to investigate the statistical properties of those values that we so far just have called ‘sources of error’. At least in criminal proceedings, the mean (or median) of the reaction time does not suffice as a base of judgement. If the vehicle of the defendant did not leave tire marks before the collision, the longest acceptable reaction time will yield in the most favourable results for the defendant. This is because the longest reaction time will leave the shortest braking distance. If the vehicle has left tire marks, the shortest reaction time will favor the defendant because it will yield in the shortest defense distance. Therefore, the recommendations of the *20. Deutscher Verkehrsgerichtstag* were not limited to medians of reaction times in traffic but also included the acceptable lower and upper limits at the 2%- and 98%-percentiles of the reaction times.

Calculating these percentiles, one usually does not distinct between intra- and interpersonal differences. This is justified by experimental as well as juridical reasons:

Concerning the evaluation of the experiment, we face the practical problem that stable estimates of the individual 2%- and 98%-percentils cannot be gained by such few measurements as for the median (= 50%-percentile). To estimate these quantities, we would need much more measurements for each test person at each combination of test factors. But if we combine all measurements concerning one combination of test factors for all test persons to one poole of measurements we will need much less effort to gain stable estimates for these percentiles.

Additionally, we have to keep in mind that our experimental setup only evaluates the short time variations of intrapersonal performance. To gain the complete bandwidth of individual variations of performance, we would need multiple investigations on one and the same person accounting for time of day, attitude, training etc. Only such a thorough investigations on intrapersonal variances may enable us to segregate it from interpersonal variances.

From the juridical point of view, the distinction between intra- and interpersonal variances in reaction time is of minor interest because the individual reaction performance of the defendant is seldomly examined. Under this condition, we would anyway have to apply the overall distribution to the defensive reaction time. (Although one may ask if the 98%-percentil of the overall population really is adequate for a motorcycle driver with ‘sporty ambition’.)

4.4 Distribution Density Function

According to the arguments mentioned above, specifying the percentiles of the overall distribution will suffice the needs of jurisdiction. But we will face new problems if there are particulars in the real accident that are not mirrored by the experimental setup. For instance the attorney may plead that the defendant did already place his foot on the braking pedal in advance. So he will not lose the time needed to move his feet from the gas pedal to the brake pedal. Or the defendant may have had to turn his head in order to recognize a special situation. The first objection may be taken into account in the experimental setup by measuring the foot moving time. But as there are no limits to imagination we will often face situations that were not taken into account in experimental design.

The evaluation of the percentiles is not sufficient to calculate the bandwidth of reaction time in such cases. Although you may consider the time needed to turn the head and the normal reaction time as being additive, you may not just add the corresponding percentiles of the processes ‘turning the head’ and ‘genuine reaction’.

In order to calculate the distribution function of two cascaded processes you need to know the distribution function of each simple process (and convolute them to get the overall distribution function). For that reason the recommendations of the *20. Deutscher Verkehrsgerichtstag* included the distribution functions for a lot of sub-processes such as ‘moving the focus’, ‘information processing’, ‘displacing the foot from the gas to the brake pedal’ and ‘pushing the brake pedal down to the limit’. As the recommendations assume these simple processes as cascaded and the corresponding response times as being additive, the response time of the overall process results from the convolution of the response times for the simple processes. The recommendations include some overall response times for common combinations of some of these simple processes. (Although erroneously the overall response time was not calculated by the convolution integral.) In order to enable calculations comparable to that proposed by the *20. Deutscher Verkehrsgerichtstag* we will also supply data concerning the distribution function.

For the analytical description of the distribution density function we will use a Gamma distribution with temporal offset as proposed in [4]

$$f(t)=\frac{1}{(n-1)!\cdot T_C}\left(\frac{t-t_0}{T_C}\right)^{n-1} e^{-\left(\frac{t-t_0}{T_C}\right)}$$

(In contrast to the presentation in [4], λ was substituted by the characteristic time T_C with $T_C = 1/\lambda$). The parameters t_0 , T_C and n of this functional approach were fitted such that the experimental distribution was reflected as good as possible. The parameters may be fitted with the strategies described in Appendix A.

The structure of the mathematical model should be chosen such that it is capable to reflect the most important properties of the experimental distribution. These consist in the position of the apex (mode), the width of the 'summit' (standard deviation) and the differing rising behavior left and right of the mode (skew). For each independent property of the measured distribution the analytical approach has to supply one independent parameter so that we reach at a minimum of three parameter.

4.5 Special Problems of Parametric Identification

According to the preceding sections, the measurements have to be pooled for each test factor combination to get stable estimates for the lower and upper percentiles of the distribution. But for the median (the 50% percentile of the distribution) we may draw stable estimates from the data gained for each simple test person. So we may also calculate the overall median as the mean of all single person test medians. The question arises whether we will arrive at the same results by these alternative procedures. Is the median of the pooled data equal to the mean of the medians of the single test persons? And how is the matter concerning the 5% and 95%-percentile?

From a juridical point of view this question may be of interest. Let us suppose that we have postulated the 95% -percentile of reaction time as the upper acceptable limit for the case under consideration. The evaluation of our pooled data has made sure, that in 95% of all cases people will reacted faster than that. But as an alternative to this we might also have proceeded such that we evaluated the distribution of reaction time for each test person of a representative sample of the population. By calculating the mean of the percentiles drawn from the measurements of each person we would make sure, that each (test) person contributes to this mean in its proportion of the overall population.

If we pool test data prior to evaluation we have to pose the question whether the people with slow reactions are mistreated over-proportionally, i.e. more than may be justified by there proportion of the overall population. This question may hardly be answered by intuition, because in such cases our intuition is as least skewed as the distribution function. Appendix B contains a mathematical treatment of this problem. At this point we will only present the solution:

- Pooling of the raw data will favor slow reactors, because they will rise the median and the 95%-percentile over-proportionally (i.e. more than is justified by there proportion of overall population). This statement holds for all left-steep distributions as long as the medians of different test persons are not extremely divergent (more than 0.5 s).
- On the other hand, fast reactors will lower the 5%-percentile over-proportionally. This statement holds under the limitation that distribution functions resemble that of the Gamma distribution.

The additional limitation mentioned in the second statement is irrelevant for practical use, because the interpersonal variation of the lower percentiles is much smaller than that of the upper percentiles. If the case under consideration includes skid marks, the avoidance is much less affected by the postulated reaction time.

So pooling of the raw data in enlarges the bandwidth defined by the 5%– and 95% percentiles. If jurisdiction applies values drawn from pooled data, it will be on the safe side.

4.6 Summary of Evaluation Strategies

So the raw data is evaluated three times:

- From the pooled data taken from all test persons we will draw the 2%, 5%-, 95%-, 98%- and 50%-percentile (median) as distribution independent measures for all combinations of experimental factors
- From the same database we will draw the distribution density functions of reaction time for each combination of test factors. The experimental distribution will be fitted by a three dimensional model (time-shifted Gamma distribution)
- The reaction time medians of the single test persons will be subjected to a two-factor analysis of variance (with complete resampling).

5 Results

5.1 Medians

Fig. 3 and **Table** 2 show how the median of reaction time changes with contrast and seeing angle. Both figure and table show the mean of reaction times for all test persons in comparison to the median of the pooled data.

	foveal					**periphery**				
Contrast	*0.09*	*0.19*	*0.29*	*0.4*	*0.52*	*0.09*	*0.19*	*0.29*	*0.4*	*0.52*
Mean	679	649	622	612	608	662	625	611	598	590
Standard Deviation	157	153	146	137	137	159	158	157	148	141
Skew	1.2	1.3	1.7	1.3	1.5	1.0	1.1	1.5	1.1	1.3
2%-Percentile	441	427	398	399	394	432	400	388	386	386
5%-Percentile	463	456	441	427	427	460	448	415	413	413
Median	654	624	604	593	589	644	598	589	578	572
95%-Percentile	1013	1033	899	907	891	1025	1003	975	905	874
98%-Percentile	1366	1338	1248	1228	1086	1354	1318	1209	1027	1094

Table 2: Characteristic values of the experimental data (Raw data clipped at 98%-percentile)

For the mean of the medians of reaction time the standard error may be calculated

$$\sigma_n = \frac{\sigma}{\sqrt{n}}$$

This value is displayed as the error span at each data point. The medians of the pooled data are around 10 – 20 ms lower than the means of the medians for all test persons. The cause of the effect was explained in section 4.5 (compare also appendix B).

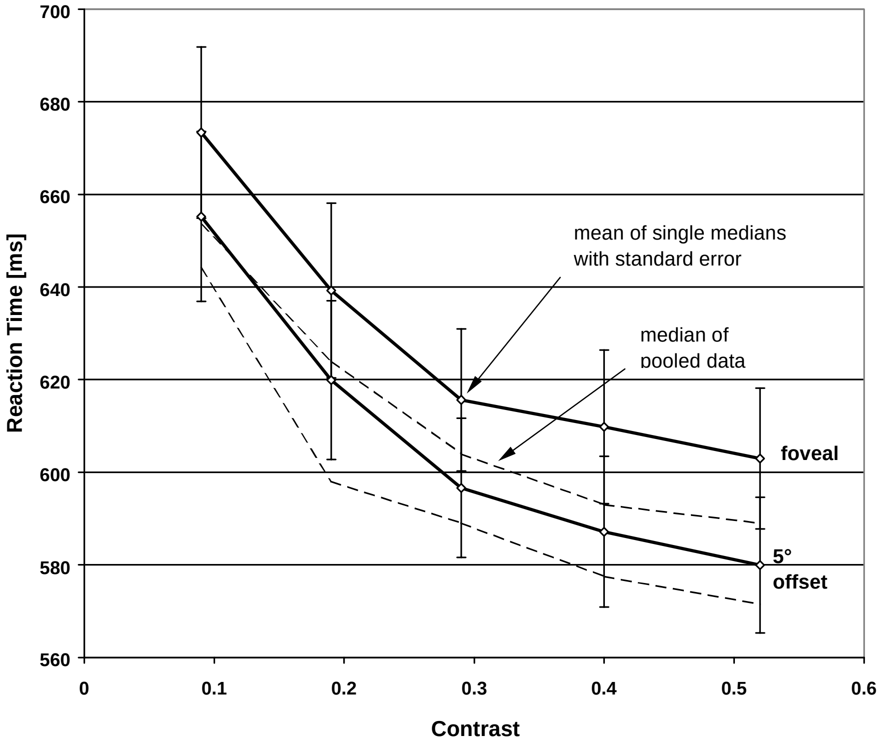

Fig. 3: Medians of the reaction times of 48 subjects

From the diagram we may draw the general tendency that reaction times rise with decreasing contrast. Approaching the threshold of perception the reaction time has to rise asymptotic versus infinity [10]. In accordance with theory the slope of the line in **Fig.** 3 rises progressively with degreasing contrast. In [4] decreasing contrast from 10.4 to 0.5 (ratio 21:1) prolonged reaction times about 40 ms as a mean. The results of this paper show that a further decrease in contrast from 0.5 – 0.09 (ratio 5.6 : 1) generated an additional rise of about 80 ms.

The lowest contrast examined in our experiments were about 4.5 times over threshold. So it lies within the range that is discussed in nocturnal pedestrian accidents (praxis factor 3 – 5). In our experimental setup this contrast prolonged reaction times about 120 ms (= 80 ms + 40 ms) in respect to their lower limit at high contrasts.

Within the reach examined by this experiment the mean of reaction times for peripheral perception lay 17 ms below that for foveal perception. This is plausible because at the background luminosity of 0.5 cd/m², faint contrasts where much more remarkable at peripheral perception. The weakest contrast displayed was hardly perceptible at foveal perception but clearly remarkable at peripheral perception.

At the highest contrast this observation contradicts the results of our first experiment, because in that experiment the reaction for foveal perception was faster. But the difference was only

13 ms and could not proved to be significant. Apart from this observation the reaction times for the lowest contrast in the last experiment are quite precisely reproduced by the result of this experiment. The mean of the median at peripheral perception this time is at 596 ms and has been at 573 ms in the first experiment. Considering foveal perception the difference between the experimental results is 45 ms.

Fig. 4 shows the same curves as **Fig.** 3 for the ipsative means, i.e. for each test person the mean of his or her medians was subtracted from all single medians and we then calculated the mean of this diminished values. The curves look almost the same as the **Fig.** 3. But as the difference in performance of the different test persons has more or less eliminated by this, the standard error is significantly lower than for the data points of **Fig.** 3.

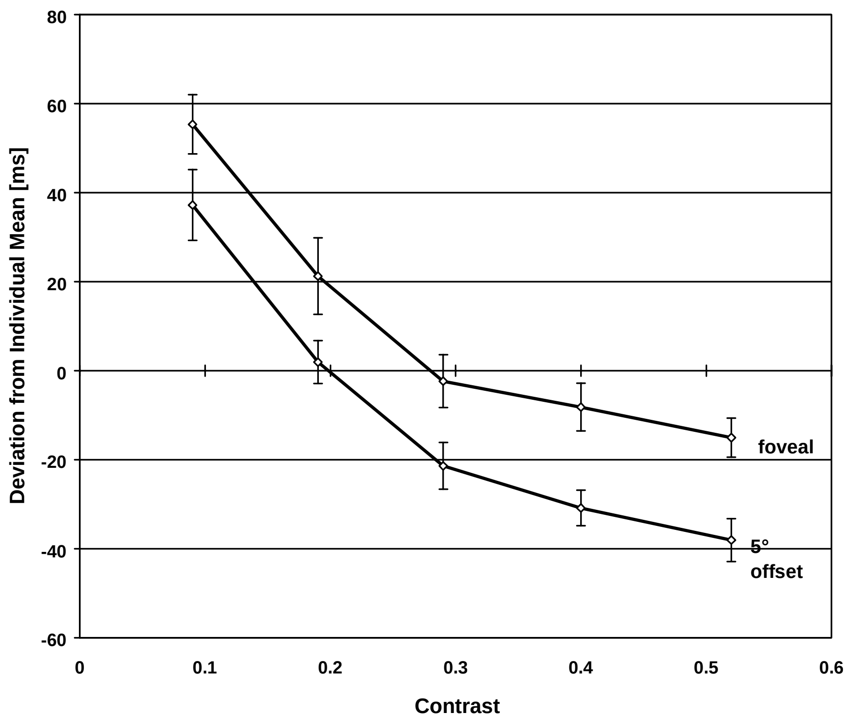

Fig. 4: Mean differences in the reaction times of 48 subjects

5.2 Percentiles

At least in criminal proceedings the median or mean of reaction time is of minor interest in comparison to the upper and lower limits that have to be taken into account. In our experiment we stated seven measurements lower than 100 ms and 12 measurements above 2500 ms. At the lower boundary we have to call in question that such short reaction times (including motion of the foot) are physiologically possible. At the upper limit we may doubt that the test person has been attentive to the experiment at that time.

We therefore have to settle statistical limits for the measurements. It is common to cut of the measurements at a given lower and upper percentile. The choice of ‘suitable’ percentiles is a duty of jurisdiction. The *20. Deutscher Verkehrsgerichtstag* has chosen the 2%- and 98%-percentile for its recommendations. But it did not use the percentiles of the original measurements but the according percentiles of the fitted distribution function. The 98%-percentile of the original measurements was significantly higher. In contrast to that, in our first paper concerning this theme we have chosen the 95%-percentile of the original data. As the percentiles are independent of distribution function, there is no good reason to draw these parameters from the fitted model, as they may be drawn from the original measurements. Our opinion is, that drawing the percentiles from the original measurements leads to a more comprehensive decision. In order to avoid anticipation of the settling by jurisdiction, **Table** 2 and **Fig.** 5 show several potential valuable percentiles.

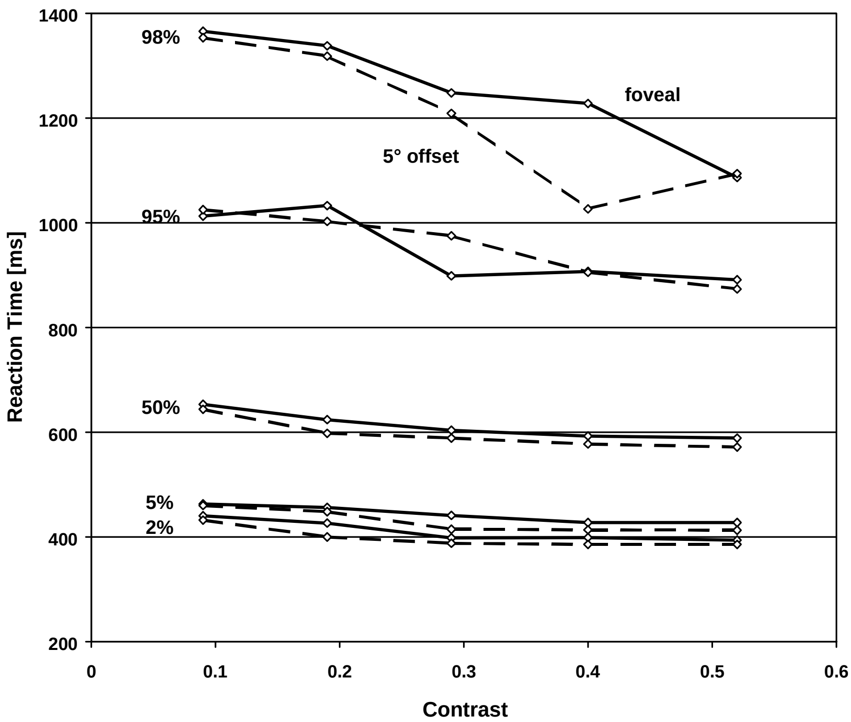

Fig. 5: Several percentiles of the pooled data

Fig. 5 shows that the upper percentiles are in further distance to the median than the lower percentiles. This effect shows again that reaction times are subjected to left-steep distributions. In the upper range of the percentiles the according reaction time is much more affected by the exact choice of the percentage than in the lower reach. For the faintest contrast the difference in reaction time between the 95%- and 98%-percentile is about 400 ms.

Interesting enough, the 95%-percentile of reaction time in our laboratory experiment is about one second. This complies with the 98%-percentile recommended by the *20. Deutscher Verkehrsgerichtstag* that has been drawn from a field experiment and that is intended to be applied to traffic accidents.

From **Table** 2 we may draw the tendency that mean and standard deviation decrease with rising contrast. This tendency for the mean is an almost inevitable consequence of the same effect observed for the medians. The increase in standard deviation while nearing the perception threshold is also a well-known effect from other reaction time experiments. As a qualitative effect it may easily be explained. As we already pointed out, the reaction time approaches infinity as we approach the perception threshold. But the exact value of the perception threshold will vary between different test persons. So test persons with weak perception abilities will be nearer to their own specific perception threshold at the lowest contrast as other persons with better perception abilities. As a consequence the reaction time of weak perceptors is already significantly raised, while this effect doesn't affect good perceptors. So the pooled data will show a growing standard deviation as contrasts get lower. Strictly speaking this effect should also apply to the skew of the distribution, but his effect is not reflected by our experimental data.

5.3 Distribution Density Functions

For each combination of experimental factors our experiment resulted in 480 measurements of reaction time. From this database we may draw a quite reliable estimate of the distribution density function. **Fig.** 6 illustrates that the analytical approach given in section 4.3 is a good description of the experimental distribution.

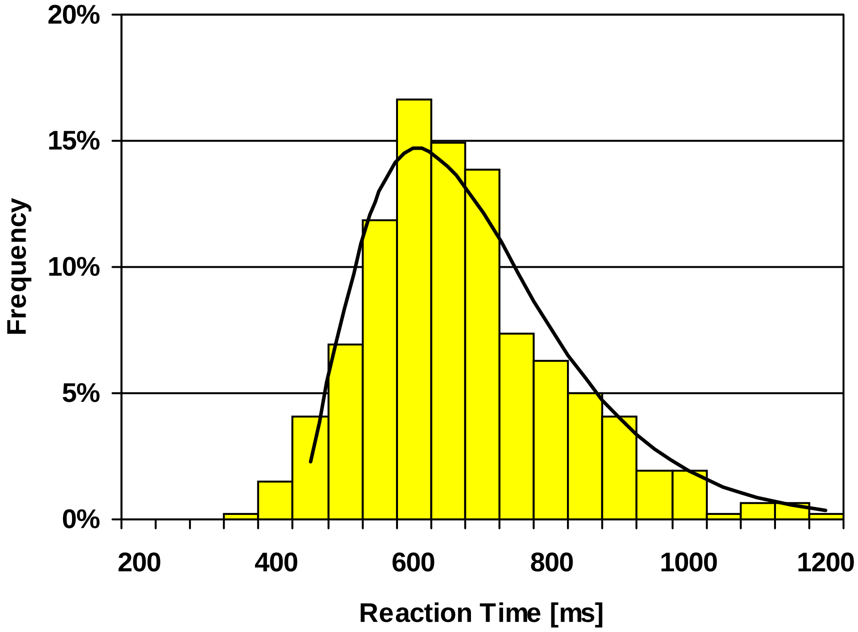

Fig. 1: Empirical histogram fitted by a time-shifted Gamma distribution (lowest contrast and foveal perception)

Table 3 lists all parameter sets for the time-shifted Gamma distributions as they were fitted to experimental data.

	foveal					periphery				
Contrast	*0.09*	*0.19*	*0.29*	*0.4*	*0.52*	*0.09*	*0.19*	*0.29*	*0.4*	*0.52*
Skew (all measurements)	3.5	7.7	19.7	4.3	5.0	3.4	21.0	10.3	1.7	4.0
Skew (clipped at 98%)	1.2	1.3	1.7	1.3	1.5	1.0	1.1	1.5	1.1	1.3
Skew (best fit)	1.1	1.0	1.0	1.0	1.1	1.2	1.1	1.1	1.1	1.1
n	3.1	4.0	4.2	4.4	3.5	2.7	3.6	3.3	3.5	3.4
T_c	89	76	71	65	73	97	84	86	79	76
t_0	417	344	325	323	352	399	327	326	322	329

Table 3: Skew of experimental data and parameters of the time-shifted Gamma distribution

Fitting the parameters to the data we used the method described in appendix A. In addition, the table shows the influence of different estimation methods on the skew of the distribution function. If the skew is calculated from the raw measurements, raw data has to be clipped at the 98 %-percentile in order to get stable estimates. If we apply the estimation method described in appendix A, the estimated skew is almost the same for all combinations of experimental factors.

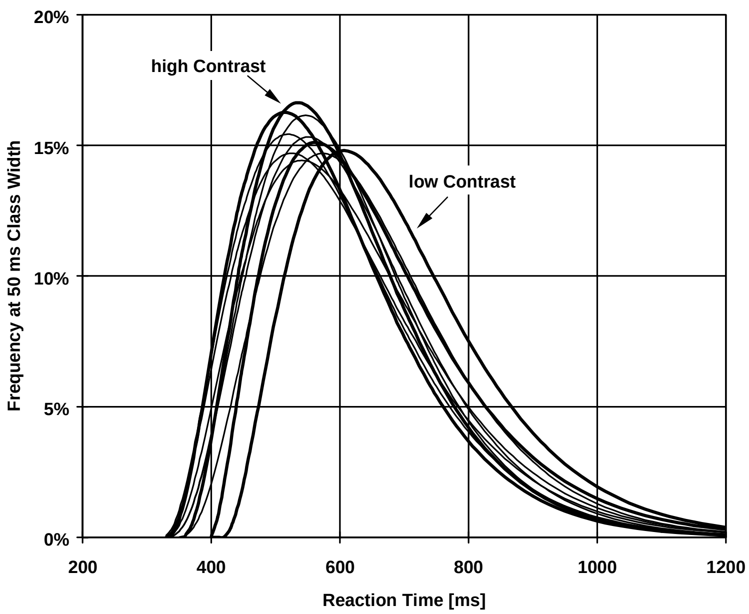

Fig. 7: Fitted density functions for the 10 treatment combinations

Fig. 7 shows the ten density functions that were fitted to the experimental distributions. The four density functions gained for the lowest and highest contrast (remember that the seeing angle did change, too) were emphasized by thicker lines. According to the observations fixed in the preceding sections, stronger contrast lead to narrower distribution functions with lower mode, while weaker contrast lead to broader density functions with greater modal parameter.

5.4 Analysis of Variance

Figures 3 and 4 show that the median of reaction time rises with decreasing contrast and that in the whole range of contrasts examined the reactions to peripheral objects were somewhat quicker than those two foveal objects. While in **Fig.** 3 the error bars for both seeing angles still overlap, the error bars on **Fig.** 4 segregate as we diminish the effect of interpersonal differences in ability. The probability that the real mean lies within the range signalized by the error bars is about 70 %.

By an variance analysis we examine with what probability the effects of the experimental factors contrast and seeing angle may just be due to hazard. As we examine one and the same sample of the overall population (the test persons) under the same experimental factors contrast and seeing angle, we apply a two factorial variance analysis for complete resampling. We regard the test persons as a third stochastic factor and result in one measurement (the median of one test person) for each combination of experimental factors.

In the statistical analysis the influence of both experimental factors could be proven highly significant:

Contrast: $F = 33{,}75 > [F_{4,\ 188,\ 99\%} = 3.42]$
Seeing angle: $F = 20{,}87 > [F_{1,\ \ 47,\ 99\%} = 7{,}21]$.

As we may expect by a look at **Fig.** 3, both experimental factors do not interact.

6 Summary and Discussion

The results presented in this paper show that the median of reaction times rises about 80 ms in the range of contrast 4 – 20 times over threshold. The median of reaction time shows the well-known exponential decrease as contrast rises [10,14]. But the range of rapidly rising reaction times is still below the contrast range examined by our experiment. From the reconstructionist's point of view the reaction times to contrast that are 4.5 times over threshold are almost identical to those to severe contrast.

Another experiment performed by Roenitzsch [11] shows, that the exponential decrease of reaction time with rising contrast still holds if the experiment consists of a discrimination task in combination with a Go-NoGo-response.

Under the mesopic seeing conditions of this experiment the reactions to peripheral objects are somewhat faster than those to foveal objects. This results from the higher sensitivity of the extra-foveal retina for objects with low luminosity. Considering sheer detection a saccade is no indispensable component of reaction. In our preliminary remarks on different reaction times we pointed out, that there are situations in real traffic were the driver has to react to all peripherally detected objects with a block breaking maneuver. In such cases we should discuss the question whether we have to allow for the time loss resulting from discrimination or even focussation. If one decides that such a time loss is not admitted due to the circumstances of the accident, some significant differences to the procedure applied up to now will result.

In comparison to real traffic the environment of the experiment is poor in stimuli. Beside the concentration task that should control the gaze of the test person, there were no other stimuli concurring for the attentiveness of the test person. This test situation can therefore hardly be compared to real traffic, in which the driver has to broaden in this visual attentiveness to the whole field of vision.

Beside the influence of contrast and seeing angle, statistical properties of the reaction time where examined. We have pointed out, that even the reaction time of one single person under almost steady conditions is not a fixed but a stochastic quantity. Especially at the upper limit we have to call for a settlement which percentile has to be applied, as even in the experiment the 95- and 98 % percentiles differed about 400 ms.

In our laboratory experiment the 95 %-percentile of reaction times was about 1 sec, the 98 %-percentile depending on contrast lay in a range of 1.1 – 1.4 sec. In contrast to that the *20. Deutscher Verkehrsgerichtstag* has fixed the upper acceptable limit of reaction times in traffic accidents at just 0.98 s. This upper limit was nominally been taken from the 98 %-percentile i.e. only two percent of all reactions should be slower and not acceptable. This result contradicts those of the experiment presented in this paper.

It has been proved that pooling of data leads to a broadening of the time gap between lower and upper bounds of reaction time. Fast reactors lower the lower percentiles over proportionally. On the other hand slow reactors rise the upper percentiles of reaction time over proportionally.

The experimental distributions could be fitted by a time-shifted Gamma distributions with as astonishing coincidence. For the fitting of modes we have proposed a simple procedure that combines the advantages of parameter estimation and procedures based on the moments of the distribution.

Appendix A

The parameter of the functional approach to reaction time distribution may be fitted to the experimental data by different methods. One suitable method is to choose parameters such that the adapted function will reproduce some general properties of the experimental distribution. Using the three parameters of our functional approach we may reflect three independent properties of the experimental distribution. As independent properties we may choose the three central moments of the distribution:

Mean:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} t_i \quad \text{(1a)} \qquad \text{res.} \qquad \mu = \int_0^\infty t\, f(t)\, dt \quad \text{(1b)}$$

Variance:

$$\hat{s}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(t_i - m)^2 \quad \text{(2a)} \qquad \text{res.} \qquad \sigma^2 = \int_0^\infty (t-\mu)^2\, f(t)\, dt \quad \text{(2b)}$$

Skew:

$$\hat{\chi} = \frac{n}{(n-1)(n-2)\cdot\sigma^3}\sum_{i=1}^{n}(t_i - \mu)^3 \quad \text{(3a)} \qquad \text{res.} \qquad \chi = \frac{1}{\sigma^3}\int_0^\infty (t-\mu)^3\, f(t)\, dt \quad \text{(3b).}$$

The equations in the first column account for the fact that mean, variance and skew are to be estimated by a random sample.

So for the Gamma distribution this results in:

$$\mu = n \cdot T_C + t_0 \quad \text{(4a)} \qquad \sigma = \sqrt{n}\cdot T_C \quad \text{(4b)} \qquad \chi = \frac{2}{\sqrt{n}} \quad \text{(4c)}$$

So the parameters may easily be calculated from the experimental data. These values are estimates for the real values. So for the parameters of the time-shifted distribution we yield in:

$$n = \frac{4}{\hat{\chi}^2} \quad \text{(5a)} \qquad T_C = \tfrac{1}{2}\hat{\sigma}\hat{\chi} \quad \text{(5b)} \qquad t_0 = \hat{\mu} - \frac{\hat{\sigma}}{2\cdot\hat{\chi}} \quad \text{(5c).}$$

With this choice of the parameters, mean, variance and skew of the experimental data are reproduced exactly. The equations show that the functional dependence between the moments and the parameters of the time-shifted Gamma distribution is rather easy. Using other functional approaches for the density function, the calculation may turn out much more difficult [1]. In some cases the equations may not allow for an analytical solution. In the following we will refer tot this procedure as the 'method of moments'.

The basic problem of the method of moments is that the estimates of the moments get more and more unreliable for moments of higher order. The moments of higher order are very sensitive to break-outs as they are typically observed in reaction time experiments. Even if experimental data is clipped at the 2 % and 98 % percentile, moments of higher order will vary significantly, **Table** 3.

Another possibility to calculate the parameters of the functional approach from the experimental data is the classical parameter estimation. To apply this method we have to divide measurements into classes of given width. In our evaluation we have used a fixed width of 50 ms.

This will yield in a distribution telling us with which probability some measurements will fall into the bounds of one of these classes, compare **Fig.** 5. The parameters of the function al approach are fitted such at the function reproduces the experimental distribution as good as possible. As an algebraic measure for as god as possible we define a loss function.

$$g(n, T_C, t_0) = \sum_{i=1}^{m} \left| p_i - f(t_i) \Delta T \right| .$$

In the equation above m stands for the amount of classes, p_i for the portion of measurements that fall in class i, Δt for the constant class width and t for the temporal mean of class i. In most cases it is important to define a loss function such that positive and negative deviations are weighted equally and positive (so-called even loss function). Therefore we take the absolute of the deviations before summarizing them. That choice of parameters that will minimize the value of the loss function and therefore approaches experimental data as god as possible is the best estimate for the parameters.

Instead of fitting the distribution density function to the experimental distribution, we may also fit the distribution function to the distribution as has been done in [1]. Luce [6] proposes the hazard function as a base for the parameter fit. In principal we might use any function that is related to the distribution functions and whose shape may be derived from the measurements as a basis for the parameter fit. The choice of a different function will only distore the scale according to which the deviations are weighted. This distortion will affect the proportion to which different parameters influence the value of the loss function. Depending on whether we choose the distribution function, the distribution density function or the hazard function for the definition of the loss function we will get other 'optimal' values for the parameters. For the parameter fits presented in this paper we have chosen the distribution density function as a basis for the comparison performed by the loss function.

If the loss function is linear in its parameters, the square of the deviations is favored above the absolute for the definition of the loss function. In this case the loss function will be differentiable and therefore enable an analytical solution of the problem. The disadvantage of square weighting the deviations is that greater deviations are weighted over proportionally. In our case the problem is non-linear in the parameters, so that it may not be solved analytically. In such cases square weighting of the deviations doesn't pay and so we have used the absolute instead to guarantee the uniform weighting of all deviations.

The minimization of the loss function may be performed automatically by common mathematical packages. We have used the Excel Add-in 'Solver' that it is distributed with Microsoft Excel. In fact we used a combination of parameter estimation and the method of moments. We accepted mean and standard deviation of the measurements as rather reliable estimates for these quantities. According to the equations 5 a – 5 c, the calculated modes will only depend on the choice of the skew under this limitation. Considering mean and standard deviation as given parameters, the value of the loss function will only depend on the skew, i.e. the loss function is a function in one variable.

Minimization of the loss function may than be performed rather easy. Either by hand by mathematical packages (for instance Solver). This combined method will preserve mean and standard deviation of the original measurements as properties of the fitted distribution function, which may be considered as an advantage.

Appendix B

In the following we will use a simple example to examine what effect pooling of data will have on value of percentiles. The percentils of the pooled data are compared to the mean of the percentiles of different test persons. In this mind experiment we will pool two measurement sets of the same size. These sets shall underlie left-steep distributions of the same shape but differ in mean, i.e. the second distribution density function is just time-shifted in respect to the first.

$f_2(t) = f_1(t - \Delta t)$.

Calculating the mean of the means will result in

$$\bar{t} = \frac{\bar{t}_1 + \bar{t}_2}{2} = \frac{\bar{t}_1 + \bar{t}_1 + \Delta t}{2} = \bar{t}_1 + \tfrac{1}{2}\Delta t \,.$$

And for the mean of the medians we will yield at

$$\overline{T} = \frac{T_1 + T_2}{2} = \frac{T_1 + T_1 + \Delta t}{2} = T_1 + \tfrac{1}{2}\Delta t \,.$$

If we pool data, the distribution density function is also just the mean of the single distribution density functions

$$f(t) = \frac{f_1(t) + f_2(t)}{2} = \frac{f_1(t) + f_1(t - \Delta t)}{2} \,.$$

And for the distribution as the integral over the distribution density we yield at an analog equation

$$F(t) = \frac{F_1(t) + F_1(t - \Delta t)}{2} \,.$$

If the offset is small,

$$F_1(t - \Delta t) = F_1(t) - f_1(t) \cdot \Delta t$$

will hold and therefore

$$F(t) = F_1(t) - \tfrac{1}{2} f_1(t) \cdot \Delta t \,.$$

Under this condition the median of the overall distribution will also not deviate much from the median t_1 of the distribution

$T = T_1 + \Delta T$,

Which will result in

$$F(T) = F(T_1) + f(T_1) \cdot \Delta T$$
$$= F_1(T_1) - \tfrac{1}{2} f_1(T_1) \cdot \Delta t + \left(f_1(T_1) - \tfrac{1}{2} \dot{f}_1(T_1) \Delta t \right) \Delta T$$

With

$$F(T) = F(T_1) = 0{,}5$$

We arrive at

$$-\tfrac{1}{2} f_1(T_1) \cdot \Delta t + \left(f_1(T_1) - \tfrac{1}{2} \dot{f}_1(T_1) \Delta t \right) \Delta T = 0 \quad ,$$

And therefore

$$\Delta T = \frac{f_1(T_1)}{f_1(T_1) - \tfrac{1}{2} \dot{f}_1(T_1) \Delta t} \cdot \tfrac{1}{2} \Delta t = -\frac{1}{1 - \frac{\dot{f}_1(T_1) \Delta t}{2 f_1(T_1)}} \cdot \tfrac{1}{2} \Delta t$$

Defining the quantity

$$\omega(t) = \frac{\dot{f}(t)}{f(t)}$$

We may put this shorter

$$\Delta T = \frac{1}{1 - \tfrac{1}{2} \omega(T_1) \, \Delta t} \cdot \tfrac{1}{2} \Delta t \, .$$

This equation compares the time shift of the median of the pooled data with the time-shift resulting from taking the mean of the medians. If $\omega(T_1)$ is positive, this time-shift will be smaller, in the other case it will be greater. As $f(t)$ is always positive, this will only depend on the value of $\dot{f}(t)$. If we have a left-steep distribution, the modal value (i.e. the maximum of f(t)) will be lower than median, so that f(t) has a falling tendency as T approaches the median. That means that $\omega(T_1) > 0$ and therefore

$$\Delta T < \tfrac{1}{2} \Delta t \, .$$

$$T = T_1 + \Delta T < T_1 + \tfrac{1}{2} \Delta t = \overline{T} \, .$$

So the median of the pooled data is smaller than mean of the single medians taken from the raw data. This result holds for all percentiles above the 50 %-percentile, because the slope of distribution density function stays negative in that range. So this result will also hold for the 95 %-percentile. E. i. the 95 %-percentile of the pooled data will lay below the mean of the 95 %-percentiles of the single distributions.

For the percentiles below the median the same question is more difficult to answer. Because the slope of the distribution density function then is positive, the denominator in our comparison equation may be affected by a chance in pre-sign. This change will arise if

$\frac{1}{2}\omega(t)\ \Delta t > 1$.

For the Gamma distribution **ω(T)** may be calculated analytically. This will yield in

$$\omega(t) = \frac{(n-1) - {}^{t}\!/_{T_C}}{{}^{t}\!/_{T_C}}$$

And therefore

$$\lim_{t \to 0} \omega(t) = \infty .$$

Considering reaction time distributions, our interest is limited to the range **n** > **2**. In that case the ratio t/T_C is greater than 0.36 (0.22) for the 5%-percentile (2%-percentile). This means that ω(T) is below 1.8 (3.6). As the offset Delta t between different test persons surely is below 1 s (0,5 s) for one and the same combination of experimental factors, we know that Omega (T) × Δt < 1. That means that the denominator will not get negative and the time shift for the pooled data is greater than that resulting from taking the mean. The situation is opposite to that of the upper percentiles, which means that deriving the upper and lower percentiles from the pooled data will result in a smaller bandwith than taking the mean of the according percentiles of different test persons.

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Erratum

We are sorry to admit that in our explanation of the observation that the mean of the medians is always greater than the median of the pooled data, we have made a mistake. It is obvious that we should expect $\Delta T \approx \frac{1}{2}\Delta t$. So we expanded F(t) and $f_1(t)$ in a Taylor series to calculate the correction terms that decide which of the terms is actually greater. We have stopped both expansions at first order terms, i.e. $f_1(T_1)$ respectively $\dot{f}_1(T_1)$:

$$\begin{aligned} F(T) &= F(T_1) + f(T_1)\cdot\Delta T \\ &= F_1(T_1) - \tfrac{1}{2} f_1(T_1)\cdot\Delta t + \left(f_1(T_1) - \tfrac{1}{2}\dot{f}_1(T_1)\Delta t\right)\Delta T \end{aligned}$$

In the case under consideration we should have taken the second order term of the first Taylor series into account, as it is of the same magnitude as the first order term of the second Taylor series:

$$\begin{aligned} & F(T) = F_1(T_1) - \tfrac{1}{2} f_1(T_1)\cdot\Delta t + \tfrac{1}{4}\dot{f}_1(T_1)\cdot\Delta t^2 + \left(f_1(T_1) - \tfrac{1}{2}\dot{f}_1(T_1)\Delta t\right)\Delta T \\ & \Rightarrow F(T) - F_1(T_1) - \tfrac{1}{2} f_1(T_1)\cdot\Delta t + f_1(T_1)\Delta T + \tfrac{1}{4}\dot{f}_1(T_1)\cdot\Delta t^2 - \tfrac{1}{2}\dot{f}_1(T_1)\Delta t\Delta T = 0 \\ & \Rightarrow \left(f_1(T_1) + \dot{f}_1(T_1)\cdot\Delta t\right)\left(\Delta T - \tfrac{1}{2}\Delta t\right) = 0 \end{aligned}$$

It turns out that the correct expansion will obstruct the way of solutions followed up by our first arguing, And if we would take higher order terms into account, the deduction would get rather complex.

In the following we will therefore present an alternative, correct deduction that has the further advantage of being much easier. The starting point of our considerations is the same as in appendix B: Two identical, left step distribution density functions, that are only time-shifted to one another.

Under this condition we might consider averaging of time values as a not quite correct method of calculating the overall distribution function, so that we will yield at a modified distribution function of **F*(t)**. This distribution function is time-shifted in respect to **F_1 (t)** with an offset ½ **Δt**, compare figure.

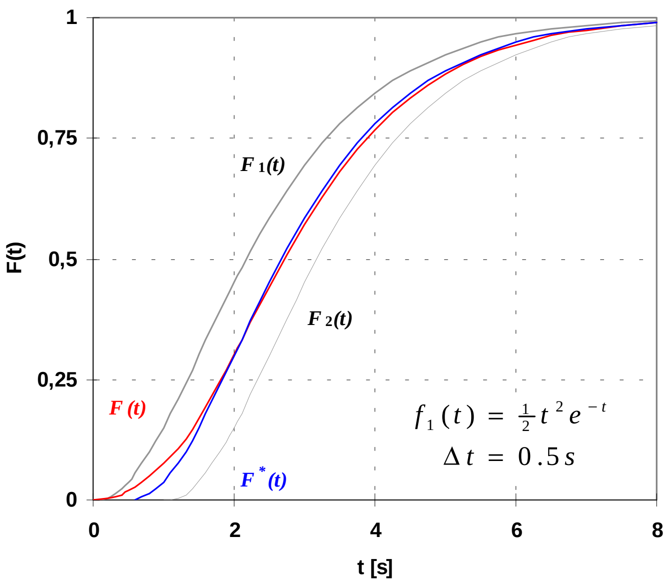

Fig. 7: Comparison between the genuine F(t) and its approximation by F*(t)
The example shows a Gamma distribution with n = 2 and λ = 1

In order to calculate the correct overall distribution function **F(t)** we have to take the mean of the function values and not the mean of the time values. If we take second order terms into account we will yield at the following nearing equations of F (t) and **F* (t)**

$$F^*(t) = F_1(t) - f_1(t)\cdot\tfrac{1}{2}\Delta t + \tfrac{1}{2}\dot{f}_1(t)\cdot\left(\tfrac{1}{2}\Delta t\right)^2$$

$$F(t) = F_1(t) + \tfrac{1}{2}\left(- f_1(t)\cdot \Delta t + \tfrac{1}{2}\dot{f}_1(t)\cdot (\Delta t)^2\right)$$

Taking the difference therefore will result in

$$F(t) - F^*(t) = \tfrac{1}{8}\dot{f}_1(t)\cdot (\Delta t)^2$$

In the near range of the median we will have

$$\dot{f}_1(t) < 0$$

and therefore

$$F^*(t) > F(t)$$

for the lower percentiles we will arrive at the contrary.

As $F^*(t)$ lies above $F(t)$ in the range of upper percentiles, the according percentiles will be reached 'later' by $F(t)$, i.e. the upper percentiles of $F(t)$ will lie above the according percentiles of $F^*(t)$. The opposite will hold for the lower percentiles, so that the range bounded by given percentiles of $F^*(t)$ will enclose the according range of $F(t)$.

So with our mind experiment we will arrive at the exact contrary of the conclusions made in the original paper: Slow reactors will be handicapped over-proportionally by pooling the measurements. As $\dot{f}_1(t)$ is very small in range of upper percentiles, this handicap is not worth mentioning. In the range of lower percentiles $\dot{f}_1(t)$ may be a considerable quantity but the possible variations is comparably small in this range, so that effect does not matter anyway.

The result of our mind experiment therefore seems to contradict the result that may be drawn from the experiment, because in that case taking the mean of the medians really resulted in greater values than taking the median of pooled data. This apparent contradiction is dissolved instantly, if we take into account that the medians drawn from the experiment were based on only ten measurements, while in our mind experiment we started from the limes of a steady distribution function.

So in a second mind experiment we suppose a stochastic process with left-steep distribution function from which we will draw a lot of samples of size N. From each of the samples we calculate the median and than calculate the mean of these medians in a second step. For the case $N = 1$ the 'average mean' will be identical to the mean of the overall distribution. (The same will hold for $N = 2$, because the median of two data points is identical to the mean.) If N is equal to the amount of measurements in the overall sample, the average median will be equal to the median of the overall sample.

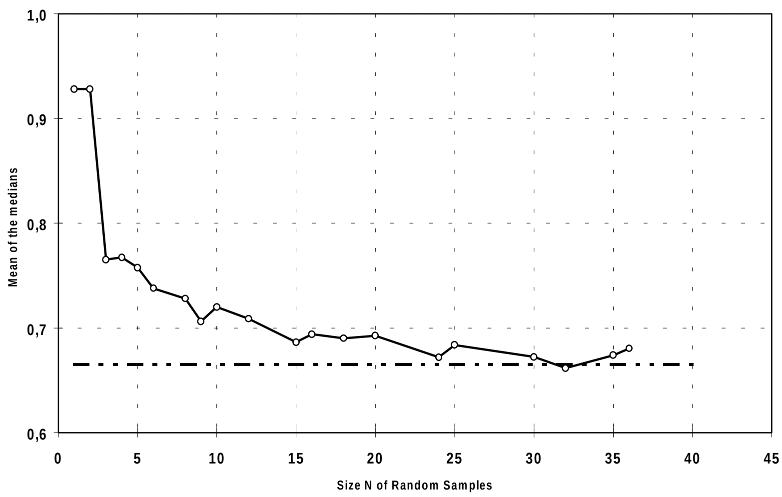

Fig 8: Mean of medians for random samples of varying size

As a median of a left-steep distribution lies below the mean, we will state a steady decrease of our average median as N rises, until we reach the median of the overall sample for large N. **Fig. 7** illustrates this effect on an exponentially distributed sample with 720 measurements.

For the distribution density function, mean $\bar{t}$ and median T we get

$$f(t) = \tfrac{1}{T_C} e^{-t/T_C} \qquad ; \qquad \bar{t} = T_C \qquad ; \qquad T = \ln(2) .$$

We have chosen $T_C = 1$, so for a sample of infinite size the mean is 1 and the median as 0.693. For the overall random sample consisting of 720 measurements the actual values were:

Mean:	0.928
Median:	0.665

The median of the sample is indicated in the figure by the dash-dotted line.

As can be drawn from the figure, the function starts at the mean for N = 1 or 2 and stochastically approaches the mean for larger values of N. In contrast to this mind experiment we may not consider the measurements for the different test persons as being taken from the same underline distribution function. But the effect illustrated by **Fig. 7** will still hold under this somewhat deviating situation.